Oscar's Physics Phaire
A Collection of Problems

You will find here a number of (mostly) elementary physics problems dealing mainly with uniform motion kinematics. In preparing this collection I have tried to create original situations that could help bring motivation to an introductory course. You are welcome to suggest improvements and ... provide solutions!

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The Four Spiders

Four spiders move in straight lines away from a common origin on a plane in such a way that at any time they are situated on the corners of a rectangle. If three spiders move at rates of 2, 3 and 4 cm/s, find out the possible values of the speed of the fourth spider.

Dangerous Nanny

On his everyday commute from home to work at 50 km/h, a man always met his nanny, heading to his house, halfway on his journey to work. One day the man left home 5 minutes later and had to drive at 70 km/h in order to get to work on time. On this day, he met his nanny 9 km from his house. What is the speed the nanny used to maintain on her daily ride?
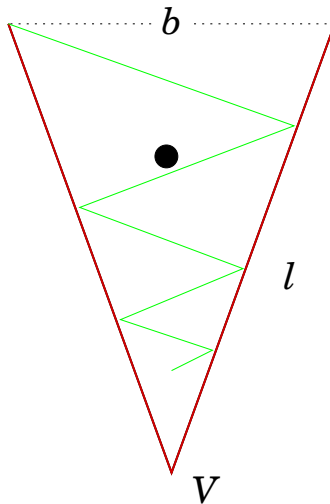
A Sliding Problem

A bead slides with constant speed v along lines perpendicular to the sides of a cone of side l and base b. How long does it take for the bead to reach the vertex V?

Cubic River

A boat that can move at speed v in still water crosses a river of width l that flows with speed w. Prove that the shortest time the boat takes to complete its trip is l/v. Show that the boat reaches the opposite bank at a distance $x = wl/v$ (above or below?) from its starting position. Verify that this value for x is a root of the equation

$$v^2 x^6 + l^2(2v^2 - w^2)x^4 + l^4(v^2 - 2w^2)x^2 - w^2 l^6 = 0.$$

Solve this (cubic) equation for other roots and interpret your result.
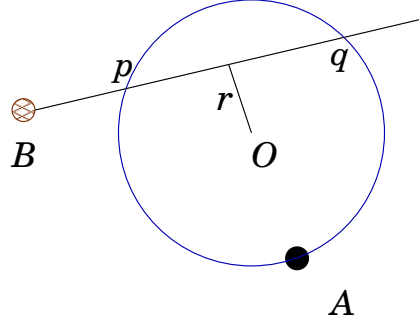
Bugs' Lives

A bug A that was confined to live on the rim of a circle of radius R realized that a tasty bug B it preyed on used to sneak into its territory always with the same constant velocity v along a same straight line located at a distance r from the enter of the circle. Bug A would like to figure out the minimum velocity it should start moving as soon as bug B showed up at p so that, no matter where it was on the circumference of the circle, it would always arrive at q before bug B, and, hopefully, have a good snack. Could you help?

The Closer the Faster

Two particles describe a rectilinear motion in a plane in such a way that the component of their relative velocity along the straight line joining them has a constant magnitude u. Prove that their relative velocity as a function of the distance d between them is given by

$$V = \frac{u}{\sqrt{1 - (d_{min}/d)^2}} \tag{1}$$

where d_{min} is the minimum distance between the particles
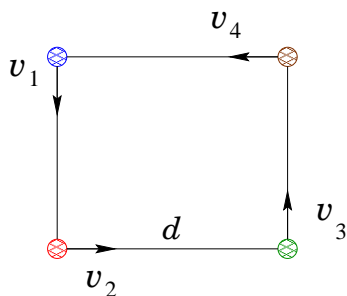
Square Dance

Four ants, initially located at the corners of a square of side d, start to move at the same time, in the same sense and with constant speeds, along the sides of the square. How long does it take for the ants to be, for the first time, all moving on the same side of the square? Are there conditions on the speeds for this to be possible?

Probabilistic Kinematics

Particles are created at random on a unit line AB and move towards B with speed v. What is the probable elapsed time between creation and detection at B?

Storm Ahead

Forty minutes into its straight line flight of 1,800 miles to Clear Skies City at 360 miles an hour, an airplane is warned against a storm ahead and ordered to take an alternative route and increase its speed 30%. Determine how far from its routine route can the plane get without delaying itself.
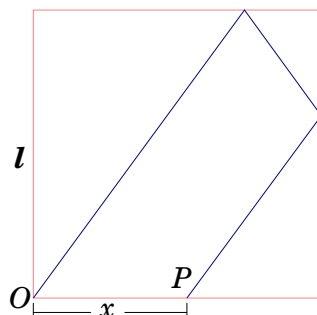
Of Life and Death

A photon, created at P with speed c, reflects twice on the walls of a square box before being absorbed at the origin O. How long does the photon survive?

Squirrels Are Not Pets

A very fearful squirrel is careful enough never to go farther than a distance R from its burrow. One day the squirrel leaves its burrow, gets a cone near a pine tree, gets a nut near an oak tree and returns to the burrow leaving behind a path in the shape of a triangle having maximum area. If the squirrel can run with speed v, how much time does it spend on the round-trip?

Fast and First

Two particles, initially at a distance r apart, move for a time T with speeds v and w along straight lines towards the same point. Determine all the points that the faster particle reaches before the slower one.

Askance and ...

Two particles that move in a plane along intersecting straight lines with constant speeds v and w have the component of these velocities along the line joining the particles always in the ratio 1:k. Prove that angle β between the directions of motion of the two particles is given by

$$\cos\beta = \frac{v^2 - kw^2}{(1-k)vw}$$

... Oblique

The components of the acceleration of a particle along two direction in the plane of the acceleration vector that make an angle ξ with each other are $a_1 = ap$ and $a_2 = aq$, where a is the magnitude of the acceleration vector. Show that

$$\sin\xi = \frac{1}{2pq}\sqrt{2(p^2 + q^2) - (p^2 - q^2)^2 - 1}.$$

Are there restrictions on the values of p, q?

Under Arrest

When the police arrived at a bank responding to a robbery, the robbers had already fled in two cars speeding at 80 km/h and 90 km/h. The police pursue the cars, arresting the slower car at 20 km from the bank and the faster car at 30 km from the bank. Determine the speed of the police car in the pursuit.
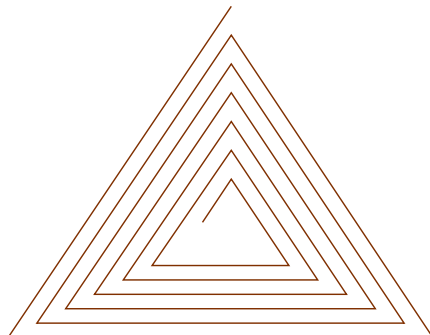
Math Illusion

A particle moves with speed v on concentric triangular lines having lengths $p\%$ shorter than the previous one. If the longest line has unit length, how long does it take for the particle to reach the center?

Downstreaming

A boat capable of developing speed v in still water crosses a river of width l and reaches the opposite shore at a distance x below its starting position. Determine the speed of the current that minimizes the time of travel.

Here and There

A particle moves in the plane defined by two straight lines that meet at an angle ψ. If the distances of the particle to both lines are always in the ratio $1:r$, show that the components of its velocity along the two lines are in the ratio

$$\frac{r + \cos\xi}{1 + r\cos\xi}$$
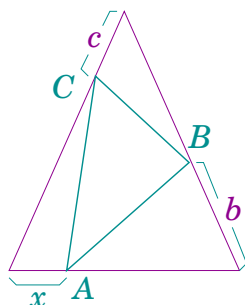
Work Out

You are willing to burn some calories *(without much effort)* and decide to run from some point A to B, C and back to A, all on the side of an equilateral triangle of length l. Before starting, find out which initial position x minimizes your route.

A Train Breakdown

The trains that serve two stations 60 km apart leave at regular intervals of 20 min. One day, one of the trains experiences mechanical problems after traveling 40 km and has to complete the rest of the trip at half its usual speed. As a consequence, this train arrives at the other station just 8 min before the next train. Determine the usual speed of the trains.

Animal Procession

A hen, a pig, and a dog leave a barn at equal time intervals and with speeds p, q, r and arrive at the farmhouse also at equal time intervals but in inverse order. Show that

$$\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$$
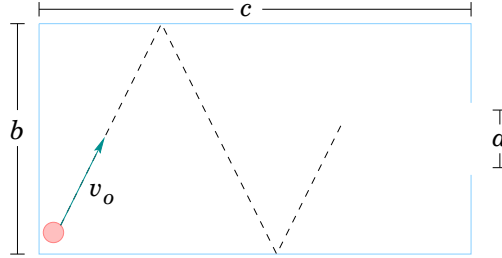
Bouncing Molecule

An air molecule on the corner of a container makes its way out of the container by progressively bouncing off its walls until it escapes through the opening. Determine the initial directions of the molecule's velocity vector that allow an escape in the case the molecule experiences no loss of speed. How would you change your answer if the molecule started off with speed v_0 and lost one third of it after each strike on the wall? What is the minimum possible value of v_0 in this case?

Back and Forth

A particle moves back and forth with speed v on a straight line OA of length p, while a second particle moves back and forth with speed w on a straight line OB of length q. If both particles start from O at the same time, how long does it take for them to cross, simultaneous and for the first time, a circle of radius r and center O in the plane AOB when

a. both particles move toward O?

b. one particle (pick one) moves toward O, the other moves away from it?

c. both particles move away from O?